



**1702EC401 – Signals and Systems**

Academic Year :	<b>2018-2019</b>	<b>Question Bank</b>	Programme:	B.E – ECE
Year / Semester:	<i>II / IV</i>		Course Coordinator:	R.KEERTHIKA

**PART – A ( 2 Mark Questions With Key)**

<b>S. No</b>	<b>Questions</b>	<b>Mark</b>	<b>COs</b>	<b>BT</b>
<b>UNIT III</b>				
1	<p><b>Why CT signals are represented by samples?</b></p> <p><math>\frac{3}{4}</math> A CT signal can not be processed in the digital processor or computer.</p> <p><math>\frac{3}{4}</math> To enable the digital transmission of CT signals.</p>	2	3	K1
2	<p><b>What is meant by sampling?</b></p> <p>A sampling is a process by which a CT signal is converted into a sequence of discrete samples with each sample representing the amplitude of the signal at the particular instant of time.</p>	2	3	K1
3	<p><b>State Sampling theorem.</b></p> <p>A band limited signal of finite energy, which has no frequency components higher than the <math>W</math> hertz, is completely described by specifying the values of the signal at the instant of timeseparated by <math>1/2W</math> seconds and</p> <p>A band limited signal of finite energy, which has no frequency components higher than the <math>W</math>hertz, is completely recovered from the knowledge of its samples taken at the rate of <math>2W</math>samples per second.</p>	2	3	K1
4	<p><b>What is meant by aliasing?</b></p> <p>When the high frequency interferes with low frequency and appears as Low then the phenomenon is called aliasing.</p>	2	3	K1
5	<p><b>What are the effects aliasing?</b></p> <p>Since the high frequency interferes with low frequency then the distortion is generated.</p> <p>The data is lost and it cannot be recovered.</p>	2	3	K1



6	<b>How the aliasing process is eliminated.</b>	2	3	K1
	i). Sampling rate $f_s \geq 2W$ . ii). strictly band limit the signal to 'W'. This can be obtained by using the Low pass filter before the sampling process. It is also called as ant aliasing filter.			
7	<b>Define Nyquist rate. and Nyquist interval.</b>	2	3	K1
	$\frac{3}{4}$ When the sampling rate becomes exactly equal to '2W' samples/sec, for a given bandwidth of W hertz, then it is called Nyquist rate. $\frac{3}{4}$ Nyquist interval is the time interval between any two adjacent samples. Nyquist rate = $2W$ Hz $\frac{3}{4}$ Nyquist interval = $1/2W$ seconds.			
8	<b>Define sampling of band pass signals.</b>	2	3	K1
	A band pass signal $x(t)$ whose maximum bandwidth is '2W' can be completely represented into and recovered from its samples, if it is sampled at the minimum rate of twice the band width.			
9	<b>Define Z transform.</b>	2	3	K1
	The Z transform of a discrete time signal $x[n]$ is denoted by $X(z)$ and it is given as $X(z) = \sum_{n=-\infty}^{+\infty} x[n] z^{-n}$ and the value n range from - to + . Here 'z' is the complex variable. This Z transform is also called as bilateral or two sided Z transform.			
10	<b>What are the two types of Z transform?</b>	2	3	K1
	(i) Unilateral Z transforms (ii) Bilateral Z transforms			
11	<b>Define unilateral Z transform.</b>	2	3	K2
	The unilateral Z transform of signal $x[n]$ is given as $X(z) = \sum_{n=0}^{\infty} x[n] z^{-n}$ The unilateral and bilateral Z transforms are same for causal signals.			
12	<b>What is region of Convergence?</b>	2	3	K1
	The region of convergence or ROC is specified for Z transform, where it Converges.			
13	<b>What are the Properties of ROC?</b>	2	3	K1
	The ROC of a finite duration sequence includes the entire z- plane, except $z=0$ and $ z =1$ .			



	<p>ROC does not contain any poles.</p> <p>ROC is the ring in the z-plane centered about origin.</p> <p>ROC of causal sequence (right handed sequence) is of the form <math> z  &gt; r</math>. v. ROC of left handed sequence is of the form <math> z  &lt; r</math>.</p> <p>ROC of two sided sequence is the concentric ring in the z plane.</p>			
14	<p><b>What is the time shifting property of Z transform?</b></p> <p><math>x[n] \leftrightarrow X(Z)</math> then</p> <p><math>x[n-k] \leftrightarrow Z^{-k} X[Z]</math>.</p>	2	3	K1
15	<p><b>What is the differentiation property in Z domain?</b></p> <p><math>x[n] \leftrightarrow X(Z)</math> then</p> <p><math>nx[n] \leftrightarrow -z \frac{d}{dz} \{X[Z]\}</math>.</p>	2	3	K2

**PART – B (12 Mark Questions with Key)**

S. No	Questions	Mark	CO	BTL
1	<p><b>State and prove the properties of Convolution.</b></p>	12	3	K2
	<p>②</p> <p><b>4.5 Properties of Convolution</b></p> <p>Let us consider two signals <math>x_1(t)</math> and <math>x_2(t)</math>. The convolution of two signals <math>x_1(t)</math> and <math>x_2(t)</math> is given by the equation</p> $x_1(t) * x_2(t) = \int_{-\infty}^{\infty} x_1(\tau) x_2(t - \tau) d\tau \quad (4.46)$ <p><b>1. Commutative property:</b></p> <p>Convolution obeys commutative property. That is</p> $x_1(t) * x_2(t) = x_2(t) * x_1(t) \quad (4.4)$			



**proof**

We have

$$x_1(t) * x_2(t) = \int_{-\infty}^{\infty} x_1(\tau) x_2(t - \tau) d\tau$$

$$\begin{aligned} \text{Let } t - \tau &= p \\ \text{then } -d\tau &= dp \end{aligned}$$

Substituting these values in Eq. (4.48) we get

$$\begin{aligned} x_1(t) * x_2(t) &= - \int_{\infty}^{-\infty} x_2(p) x_1(t - p) dp \\ &= \int_{-\infty}^{\infty} x_2(p) x_1(t - p) dp \\ &= x_2(t) * x_1(t) \\ \Rightarrow \boxed{x_1(t) * x_2(t) &= x_2(t) * x_1(t)} \end{aligned}$$

**2. Distributive property:**

$$x_1(t) * [x_2(t) + x_3(t)] = x_1(t) * x_2(t) + x_1(t) * x_3(t)$$

**3. Associative property:**

$$x_1(t) * [x_2(t) * x_3(t)] = [x_1(t) * x_2(t)] * x_3(t)$$

**4. Shift property:**

$$\text{If } x_1(t) * x_2(t) = z(t)$$

then

$$x_1(t) * x_2(t - T) = z(t - T)$$

**Proof**

$$\begin{aligned} x_1(t) * x_2(t - T) &= \int_{-\infty}^{\infty} x_1(\tau) x_2(t - T - \tau) d\tau \\ &= z(t - T) \end{aligned}$$



Similarly

$$x_1(t - T) * x_2(t) = z(t - T) \quad (4.54)$$

$$\text{and } x_1(t - T_1) * x_2(t - T_2) = z(t - T_1 - T_2) \quad (4.55)$$

### 5. Convolution with an impulse:

Convolution of a signal  $x(t)$  with a unit impulse is the signal  $x(t)$  itself.

That is

$$x(t) * \delta(t) = x(t) \quad (4.56)$$

*Proof*

$$x(t) * \delta(t) = \int_{-\infty}^{\infty} x(\tau) \delta(t - \tau) d\tau$$

$$\begin{aligned} \delta(t - \tau) &= 1 \text{ for } t = \tau \\ &= 0 \text{ otherwise} \end{aligned}$$

$$= x(t)$$

### 6. Convolution with shifted impulse

Convolution of a signal  $x(t)$  with shifted impulse  $\delta(t - t_0)$  is equal to  $x(t - t_0)$ . That is

$$x(t) * \delta(t - t_0) = x(t - t_0) \quad (4.57)$$

*Proof*

$$x(t) * \delta(t - t_0) = \int_{-\infty}^{\infty} x(\tau) \delta(t - \tau - t_0) d\tau$$
$$= x(\tau) |_{\tau=t-t_0} = x(t - t_0)$$

### 7. Convolution with unit step

Convolution of a signal  $x(t)$  with unit step signal  $u(t)$  is given by

$$x(t) * u(t) = \int_{-\infty}^t x(\tau) d\tau \quad (4.58)$$



**Proof**

$$x(t) * u(t) = \int_{-\infty}^{\infty} x(\tau)u(t - \tau)d\tau$$
$$= \int_{-\infty}^t x(\tau)d\tau \quad \boxed{u(t - \tau) = 1 \text{ for } \tau < t = 0 \text{ for } \tau > t}$$

**8. Convolution with shifted unit step**

Convolution of a signal  $x(t)$  with shifted unit step signal is given by

$$x(t) * u(t - t_0) = \int_{-\infty}^{t-t_0} x(\tau)d\tau \quad (4.59)$$

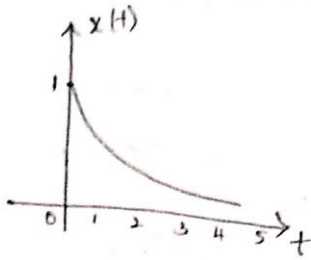
**9. Width property:**

Let the duration  $x_1(t)$  and  $x_2(t)$  are  $T_1$  and  $T_2$  respectively. Then the duration of the signal obtained by convolving  $x_1(t)$  and  $x_2(t)$  is  $T_1 + T_2$ .

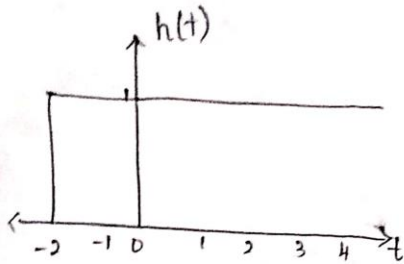
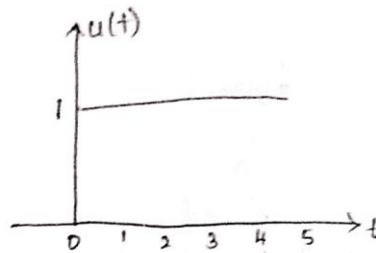
2	Find the convolution of the following signals $x(t) = e^{-2t} u(t)$ & $h(t) = u(t+2)$	12	3	K2
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Solution:

$$x(t) = e^{-\alpha t} u(t)$$



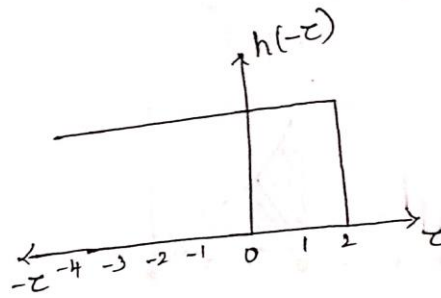
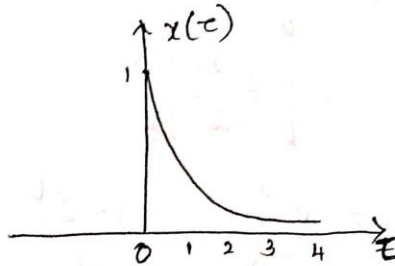
$$h(t) = u(t+2)$$



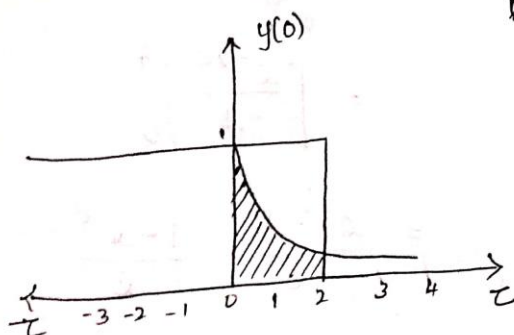
put  $t=0$

$$y(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau) d\tau$$

$$y(0) = \int_{-\infty}^{\infty} x(\tau) h(-\tau) d\tau$$



$$y(0) = \int_{-\infty}^{\infty} e^{-\alpha \tau} u(\tau) u(-\tau+2) d\tau$$



$$u(-\tau+2) = 1$$

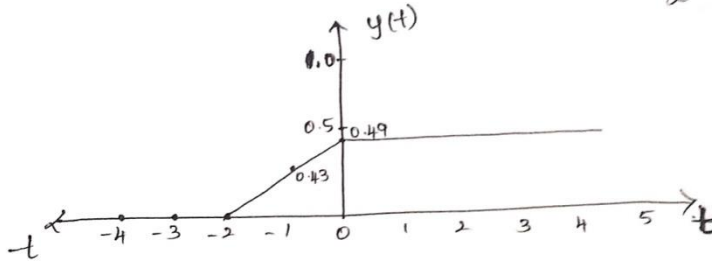
$$= \int_0^2 e^{-\alpha \tau} d\tau$$

$$= \left[ \frac{e^{-\alpha \tau}}{-\alpha} \right]_0^2$$

$$= \frac{e^{-4} - 1}{-\alpha} \Rightarrow \frac{1 - e^{-4}}{\alpha}$$



$$y(t) = \begin{cases} 0, & t \leq -2 \\ \frac{1-e^{-2t}}{2}, & t = -1 \\ \frac{1-e^{-2(t+2)}}{2}, & t \geq 0 \end{cases}$$
$$\frac{1-e^{-2}}{2} = 0.43$$
$$\frac{1-e^{-4}}{2} = 0.49$$
$$\frac{1-e^{-6}}{2} = 0.49$$





put  $t = -1 \Rightarrow \int_{-\infty}^{\infty} x(\tau) h(-1-\tau)$

$$y(-1) = \int_0^1 e^{-2\tau} d\tau$$

$$= \left[ \frac{e^{-2\tau}}{-2} \right]_0^1$$

$$= \frac{e^{-2} - 1}{-2} = \frac{1 - e^{-2}}{2}$$
  

put  $t = -2 \Rightarrow \int_{-\infty}^{\infty} x(\tau) h(-2-\tau)$

$$y(-2) = 0$$

$$y(-3) = 0$$

$$y(-4) = 0$$
  

put  $t = 1 \Rightarrow \int_{-\infty}^{\infty} x(\tau) h(1-\tau)$

$$y(1) = \int_0^3 e^{-2\tau} d\tau$$

$$= \left[ \frac{e^{-2\tau}}{-2} \right]_0^3$$

$$= \frac{e^{-6} - 1}{-2} = \frac{1 - e^{-6}}{2}$$
  

put  $t = 2 \Rightarrow \int_{-\infty}^{\infty} x(\tau) h(2-\tau)$

$$y(2) = \int_0^4 e^{-2\tau} d\tau$$

$$= \left[ \frac{e^{-2\tau}}{-2} \right]_0^4$$

$$= \frac{e^{-8} - 1}{-2} = \frac{1 - e^{-8}}{2}$$

3 Using Laplace transform, solve the following differential equations.  
 $d^3y(t)/dt^3 + 7d^2y(t)/dt^2 + 16dy(t)/dt + 12y(t) = x(t)$  if  $dy(0^-)/dt = 0$ ,  $d^2y(0^-)/dt^2 = 0$ ,  $y(0^-) = 0$   
 &  $x(t) = \mathcal{Z}(t)$

12    3    K2



$$(ii) \quad \frac{d^3 y(t)}{dt^3} + 7 \frac{d^2 y(t)}{dt^2} + 16 \frac{dy(t)}{dt} + 12y(t) = x(t)$$

Applying Laplace transform on both sides we get

$$[s^3 Y(s) - s^2 y(0^-) - s \frac{dy(0^-)}{dt} - \frac{d^2 y(0^-)}{dt^2}] + 7[s^2 Y(s) - sy(0^-) - \frac{dy(0^-)}{dt}] + 16[sY(s) - y(0^-)] + 12Y(s) = X(s) \quad (6.73)$$

Given

$$y(0^-) = 0$$
$$\frac{dy(0^-)}{dt} = 0 \quad \text{and} \quad \frac{d^2 y(0^-)}{dt^2} = 0$$

Substituting these values in Eq. (6.73), we get

$$s^3 Y(s) + 7s^2 Y(s) + 16sY(s) + 12Y(s) = X(s)$$

$$Y(s)[s^3 + 7s^2 + 16s + 12] = X(s)$$

$$\frac{Y(s)}{X(s)} = \frac{1}{s^3 + 7s^2 + 16s + 12}$$

For  $x(t) = \delta(t); X(s) = 1$ . Therefore

$$Y(s) = \frac{1}{s^3 + 7s^2 + 16s + 12}$$

$$= \frac{1}{(s+3)(s+2)^2}$$

$$= \frac{A}{s+2} + \frac{B}{(s+2)^2} + \frac{C}{s+3}$$

$$A = \frac{1}{1!} \frac{d}{ds} \left[ (s+2)^2 \frac{1}{(s+3)(s+2)^2} \right] \Big|_{s=-2}$$

$$-3 \begin{array}{ccc|c} 1 & 7 & 16 & 12 \\ 0 & -3 & -12 & -12 \\ \hline 1 & 4 & 4 & 0 \end{array}$$

$$\Rightarrow s^3 + 7s^2 + 6s + 12$$
$$= (s+3)(s^2 + 4s + 4)$$
$$= (s+3)(s+2)^2$$

$$= \frac{d}{ds} \left[ \frac{1}{s+3} \right] \Big|_{s=-2}$$

$$= \frac{-1}{(s+3)^2} \Big|_{s=-2} = -1$$

$$B = (s+2)^2 \frac{1}{(s+3)(s+2)^2} \Big|_{s=-2} = 1$$

$$C = (s+3) \frac{1}{(s+3)(s+2)^2} \Big|_{s=-3} = 1$$

$$Y(s) = \frac{-1}{s+2} + \frac{1}{(s+2)^2} + \frac{1}{s+3}$$

$$\Rightarrow y(t) = -e^{-2t} u(t) + te^{-2t} u(t) + e^{-3t} u(t)$$



$$H(s) = \frac{10}{s^2 + 6s + 10}$$

$$\frac{Y(s)}{X(s)} = \frac{10}{s^2 + 6s + 10}$$

For an impulse  $x(t) = \delta(t); X(s) = 1$

$$\Rightarrow Y(s) = \frac{10}{s^2 + 6s + 10}$$
$$= \frac{10}{(s+3)^2 + 1^2}$$

Taking inverse Laplace transform we get

$$y(t) = 10e^{-3t} \sin t u(t)$$

(ii) For a unit step input

$$x(t) = u(t)$$

$$X(s) = \frac{1}{s}$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{10}{s^2 + 6s + 10}$$

$$Y(s) = \frac{10}{s(s^2 + 6s + 10)}$$

$$= \frac{A}{s} + \frac{Bs + C}{s^2 + 6s + 10}$$

$$\Rightarrow \frac{10}{s(s^2 + 6s + 10)} = \frac{A(s^2 + 6s + 10) + s(Bs + C)}{s(s^2 + 6s + 10)}$$

$$(A+B)s^2 + (6A+C)s + 10A = 10$$

Comparing the coefficient of  $s^2, s$  and constant we get

$$A+B=0; 6A+C=0$$

$$10A=10$$

$$A=1$$

$$B=-1$$

$$C=-6$$

$$Y(s) = \frac{1}{s} + \frac{-s-6}{s^2+6s+10}$$

$$= \frac{1}{s} - \frac{(s+6)}{s^2+6s+10}$$

$$= \frac{1}{s} - \frac{s+6}{(s+3)^2+1}$$

$$= \frac{1}{s} - \left\{ \frac{s+3}{(s+3)^2+1} + \frac{3}{(s+3)^2+1} \right\}$$

Taking inverse Laplace transform we get

$$y(t) = u(t) - \{e^{-3t} \cos t u(t) + 3e^{-3t} \sin t u(t)\}$$
$$= [1 - e^{-3t} \{\cos t + 3 \sin t\}] u(t)$$



**sin(2t) u(t)**

**Solution:**

Given

$$x(t) = \sin(2t)u(t)$$

$$x(t) = \frac{2}{s^2 + 4}$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{s + 10}{(s^2 + 3s + 2)}$$

$$\Rightarrow Y(s) = \frac{s + 10}{(s + 1)(s + 2)} \cdot \frac{2}{s^2 + 4}$$

$$= \frac{2(s + 10)}{(s + 1)(s + 2)(s^2 + 4)}$$

$$= \frac{A}{s + 1} + \frac{B}{s + 2} + \frac{Cs + D}{s^2 + 4}$$

$$\Rightarrow 2(s + 10) = A(s + 2)(s^2 + 4) + B(s + 1)(s^2 + 4) + (Cs + D)(s + 1)(s + 2)$$

$$= A(s^3 + 2s^2 + 4s + 8) + B(s^3 + s^2 + 4s + 4) +$$

$$C(s^3 + 3s^2 + 2s) + D(s^2 + 3s + 2)$$

$$= (A + B + C)s^3 + (2A + B + 3C + D)s^2$$

$$+ (4A + 4B + 2C + 3D)s + 8A + 4B + 2D$$

$$A + B + C = 0; 2A + B + 3C + D = 0; 4A + 4B + 2C + 3D = 2$$

$$8A + 4B + 2D = 20$$

Solving for A, B, C and D we get

$$A = \frac{18}{5}; B = -2; C = \frac{-8}{5}; D = \frac{-2}{5}$$

$$Y(s) = \frac{18}{5(s + 1)} - \frac{2}{s + 2} - \left( \frac{\frac{8}{5}s + \frac{2}{5}}{s^2 + 4} \right)$$

$$= \frac{18}{5(s + 1)} - \frac{2}{s + 2} - \frac{8}{5} \frac{s}{s^2 + 4} - \frac{2}{5} \frac{1}{s^2 + 4}$$

$$= \frac{18}{5(s + 1)} - \frac{2}{s + 2} - \frac{8}{5} \left( \frac{s}{s^2 + 4} \right) - \frac{1}{5} \left( \frac{2}{s^2 + 4} \right)$$

$$y(t) = \frac{18}{5}e^{-t} - 2e^{-t} - \frac{8}{5}\cos 2t - \frac{1}{5}\sin 2t$$



6 i	<b>Plot the pole – zero diagram of the following transfer functions</b> $H(S)=(s+2)/(s^2+2s+2)$ , $H(S)=(s+3)/s(s^2+4)(s+2)(s+1)$	6	3	K2
	<p>(i) Given the transfer function</p> $H(s) = \frac{s+2}{s^2+2s+2}$ <p>The zeros can be obtained by equating the numerator polynomial to zero</p> $s+2=0$ $s=-2$ <p>The poles can be obtained by equating the denominator polynomial to zero</p> $s^2+2s+2=0$ $s_{1,2} = \frac{-2 \pm \sqrt{4-8}}{2} = -1 \pm j$ <p>The poles are at <math>-1 \pm j</math> and the zero is at <math>-2</math>. The pole-zero diagram is shown in Fig. 6.18.</p> <p>(ii) Given</p> $H(s) = \frac{s+3}{s(s^2+4)(s+2)(s+1)}$ <p>The zero is at <math>s = -3</math> and the poles are at <math>s = 0; s = \pm j2; s = -2; s = -1</math></p> <p>The pole-zero diagram is shown in Fig.6.19.</p> <p>07]</p>			
6 ii	<b>For a system with transfer function <math>H(S)= (s+5)/(s^2+5s+6)</math> find the zero state response if the input <math>x(t)=e^{-3t} u(t)</math>.</b>			



**Solution:** Given

$$H(s) = \frac{s+5}{s^2+5s+6}$$

$$\frac{Y(s)}{X(s)} = \frac{s+5}{(s+2)(s+3)}$$

For

$$x(t) = e^{-3t}u(t) \quad X(s) = \frac{1}{s+3}$$

$$\Rightarrow Y(s) = \frac{s+5}{(s+2)(s+3)^2}$$

$$= \frac{A}{s+2} + \frac{B}{s+3} + \frac{C}{(s+3)^2}$$

$$A = (s+2) \frac{s+5}{(s+2)(s+3)^2} \Big|_{s=-2} = \frac{-2+5}{(-2+3)^2} = 3$$

$$B = \frac{1}{1!} \frac{d}{ds} \left[ (s+3)^2 \frac{s+5}{(s+2)(s+3)^2} \right] \Big|_{s=-3}$$
$$= \frac{(s+2) - (s+5)}{(s+2)^2} \Big|_{s=-3}$$

$$= \frac{-3}{(-3+2)^2} = -3$$

$$C = (s+3)^2 \frac{s+5}{(s+2)(s+3)^2} \Big|_{s=-3} = -2$$

$$Y(s) = \frac{3}{s+2} - \frac{3}{s+3} - \frac{2}{(s+3)^2}$$

Taking inverse Laplace transform, we have

$$y(t) = 3e^{-2t}u(t) - 3e^{-3t}u(t) - 2te^{-3t}u(t)$$

**Part C (20 Mark Questions with Key)**

- 1 Using Laplace transform, solve the following differential equations.  
 $d^2y(t)/dt^2 + 3dy(t)/dt + 2y(t) = dx(t)/dt$  if  $y(0) = 0$ ,  $dy(0)/dt = 1$  &  $x(t) = e^{-t}u(t)$

20

3

K2



Taking Laplace transform on both sides we get

$$\left[ s^2 Y(s) - sy(0^-) - \frac{dy}{dt}(0^-) \right] + 3[sY(s) - y(0^-)]$$

$$+ 2Y(s) = sX(s) - x(0^-)$$

$$y(0^-) = 2; \frac{dy(0^-)}{dt} = 1$$

$$(s^2 Y(s) - 2s - 1) + 3[sY(s) - 2] + 2Y(s) = sX(s)$$

$$Y(s)[s^2 + 3s + 2] = 2s + 7 + sX(s)$$

Given  $x(t) = e^{-t}u(t)$

$$X(s) = \frac{1}{s+1}$$

$$\Rightarrow Y(s)[s^2 + 3s + 2] = 2s + 7 + \frac{s}{s+1} \quad 2s^2$$

$$Y(s) = \frac{2s+7}{s^2+3s+2} + \frac{s}{(s+1)(s^2+3s+2)}$$

$$= \frac{(2s+7)(s+1) + s}{(s+1)(s^2+3s+2)}$$

$$= \frac{2s^2 + 10s + 7}{(s+1)(s^2+3s+2)}$$

$$= \frac{2s^2 + 10s + 7}{(s+1)^2(s+2)}$$

$$= \frac{A}{s+1} + \frac{B}{(s+1)^2} + \frac{C}{s+2}$$

$$A = \frac{1}{1!} \frac{d}{ds} \left[ (s+1)^2 \frac{2s^2 + 10s + 7}{(s+1)^2(s+2)} \right] \Big|_{s=-1}$$

$$= \frac{(s+2)(4s+10) - (2s^2 + 10s + 7)}{(s+2)^2} \Big|_{s=-1}$$

$$= 7$$

$$B = (s+1)^2 \frac{2s^2 + 10s + 7}{(s+1)^2(s+2)} \Big|_{s=-1} = -1$$

$$C = (s+2) \frac{2s^2 + 10s + 7}{(s+1)^2(s+2)} \Big|_{s=-2} = -5$$

$$Y(s) = \frac{7}{s+1} - \frac{1}{(s+1)^2} - \frac{5}{s+2}$$

Taking inverse Laplace transform on both sides, we get

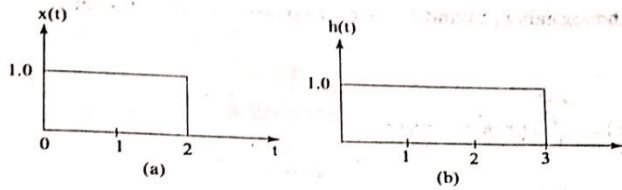
$$y(t) = 7e^{-t}u(t) - te^{-t}u(t) - 5e^{-2t}u(t)$$

2 Find the convolution of the following signals  $x(t)$  and  $h(t)$   
 $X(t) = 1$  when  $0 < t < 2$  and  $h(t) = 1$  when  $0 < t < 3$

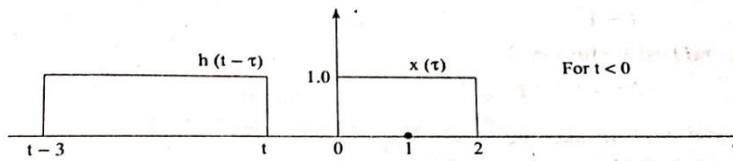
20

3

K2



**Fig. 4.12**



**Fig. 4.13**

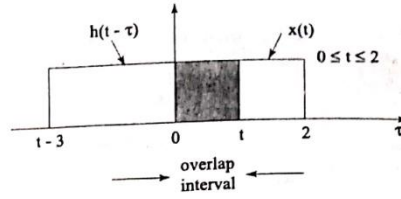
As shown in Fig. 4.13 the signals  $x(\tau)$  and  $h(t - \tau)$  does not overlap for  $t < 0$ . Therefore the product  $x(\tau) h(t - \tau)$  is zero.

That is

$$y(t) = \int_{-\infty}^{\infty} x(\tau) h(t - \tau) = 0 \quad \text{for } t < 0$$

Now shift the signal  $h(t - \tau)$  right until the right edge of the signal  $h(t - \tau)$  intersects left edge of  $x(\tau)$ .



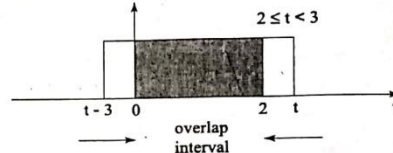


**Fig. 4.14**

In the interval  $0 \leq t < 2$

$$y(t) = \int_0^t dt = t$$

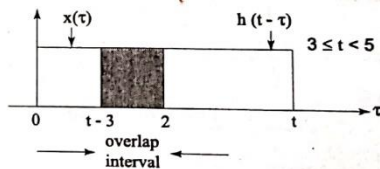
In the interval  $2 \leq t < 3$



**Fig. 4.15**

$$y(t) = \int_0^2 d\tau = 2$$

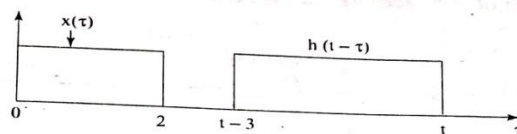
In the interval  $3 \leq t < 5$



**Fig. 4.16**

$$y(t) = \int_{t-3}^2 d\tau = 2 - (t-3) \\ = 5 - t$$

For  $t \geq 5$

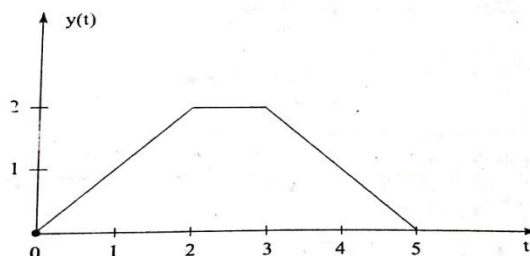


**Fig. 4.17**

The signals  $x(\tau)$  and  $h(t - \tau)$  does not overlap. Therefore  $y(t) = 0$

$$y(t) = 0 \text{ for } t < 0 \\ = t \text{ for } 1 \leq t < 2 \\ = 2 \text{ for } 2 \leq t < 3 \\ = 5 - t \text{ for } 3 \leq t < 5 \\ = 0 \text{ for } t \geq 5$$

The sketch of  $y(t)$  is shown in Fig. 4.18.





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