1702EC402–Signals and Systems

Academic Year :	**2018-2019**	**Question Bank**	Programme:	B.E – ECE
Year / Semester :	*II / IV*		Course Coordinator:	R.KEERTHIKA

PART – A (2 Mark Questions With Key)

S.No	Questions	Mark	COs	BTL
	UNIT V-LINEAR TIME INVARIANT - DISCRETE TIME SYSTEMS			
1	**How unit sample response of discrete time system is defined ?**	2	5	K1
	The unit sample response of the discrete time system is output of the system is output of the system to unit sample sequence. i.e., $T[\delta(N)] = h(n)$ Also $h(n) = z^{-1\,[H(z)\}}$			
2	**If x(n) and y(n) are discrete variable functions, what is its convolution sum?**	2	5	K1
	The convolution sum = $\sum_{k=-\infty}^{\infty} x(k)y(n-k)$			
3	**A causal discrete time system is BIBO stable only if its transfer function has**	2	5	K1
	A causal discrete time system is stable if poles of its transfer function lie within the unit circle.			
4	**How z-transform is related to Fourier transform.**	2	5	K1
	Fourier transform is basically z-transform evaluated on the unit circle. i.e., $X(z)\|_{z=e^{jw}} = X(w)$ at $\|z\| = 1$			
5	**Define system function of the discrete time system.**	2	5	K1
	The system function of the discrete time system is H(z) = Y(z)/X(z) = z-transform of the output/z-transform of the in input Or H(z) = Z{h(n)} i.e, z-transform of unit sample response			
6	**A system specified by a recursive difference equation is called infinite**	2	5	K1

	impulse response system (True/False).			
	This statement is true. An IIR system can be represented by recursive difference equation.			
7	**If u(n) is the impulse response of the system, what is its step response?**	2	5	K1
	Here h(n) = u(n) and the input is x(n) = u(n) Hence output y(n) = h(n) *x(n) = u(n)* u(n)			
8	**Is the output sequence of an LTI system finite or infinite when the input x(n) is finite? Justify your answer.**	2	5	K1
	If the impulse response of the system is infinite, then output sequence is infinite even through input is finite. For example consider, Input, x(n) = δ(n) finite length 9Impulse response, h(n) = a^n u(n) Infinite length Output sequence, y(n) = h(n) * x(n) = a^n u(n) * δ(n) = a^n u(n)			
9	**Consider an LTI system with impulse response h(n) = δ (n-n_o0 for an input x(n), find Y($e^{j\Omega}$).**	2	5	K1
	Here Y($e^{j\Omega}$) is the spectrum of output. By convolution theorem we can write, Y($e^{j\Omega}$) = H($e^{j\Omega}$) X ($e^{j\Omega}$) H($e^{j\Omega}$) = DTFT {δ(n-n_o)} = $e^{j\Omega no}$ Y($e^{j\Omega}$) = $e^{-j\Omega no}$ X($e^{j\Omega}$)			
10	**Determine the system function of the discrete time system described by the difference equation.**	2	5	K1
	Y(n)-1/2y(n-1) + 1/4y(n-2) = x(n) – x(n-1) Taking z-transform of both sides, Y(z) -1/2 z^{-1}Y(z) +1/4z^{-2}Y(z) = X(z) – z^{-1} X(z) Y(z) / X(z) = 1-z^{-1} / 1-1/2z^{-1} + ¼ z^{-2} H(z) = 1-z^{-1}/1-1/2z^{-1}+1/4z^{-2}			

11	**Write the general difference equation relating input and output of a system**	2	5	K1
	$Y(n) = -\sum_{k=1}^{N} a_k\, y(n-k) + \sum_{k=0}^{M} b_k\, x(n-k)$ Here y(n-k) are previous outputs and x(n-k) are present and previous inputs.			
12	**Determine the transfer function of the system described by y(n) = ay(n-1) + x(n)**	2	5	K1
	$Y(z) = a\, z^{-1} Y(z) + X(z)$ $Y(z)\,[1-az^{-1}] = X(z)$ $H(z) = Y(z)/X(z) = 1/1-az^{-1}$			
13	**How the discrete time system is represented.**	2	5	K1
	The DT system is represented either Block diagram representation or difference equation representation			
14	**What arte the classification of the system based on unit sample response?**	2	5	K1
	a. FIR (Finite impulse Response) system. b. IIR (Infinite Impulse Response) system			
15	**What is meant by FIR system?**	2	5	K2
	If the systems have finite duration impulse response then the system is said to be FIR system			

PART – B (12 Mark Questions with Key)

S.No	Questions	Mark	CO	BTL
1	**Obtain the direct form 1 and 2 realization for the system described by the difference equation y(n)-5\6y(n-1)+1\6y(n-2)=x(n)+2x(n-1)**	12	5	K2

Solution:

Given

$$y(n) - \frac{5}{6}y(n-1) + \frac{1}{6}y(n-2) = x(n) + 2x(n-1) \quad (1$$

Taking z-transform on both sides assuming zero initial conditions, we get

$$Y(z) - \frac{5}{6}z^{-1}Y(z) + \frac{1}{6}z^{-2}Y(z) = X(z) + 2z^{-1}X(z)$$

Let

$$X(z) + 2z^{-1}X(z) = W(z) \quad ($$

hen

$$Y(z) - \frac{5}{6}z^{-1}Y(z) + \frac{1}{6}z^{-2}Y(z) = W(z)$$

or

$$Y(z) = W(z) + \frac{5}{6}z^{-1}Y(z) - \frac{1}{6}z^{-2}Y(z)$$

Realizing Eq. (10.124), we get

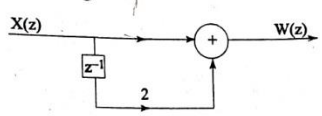


Fig. 10.22 Realization of Eq. (10.124).

Similarly realization of Eq. (10.125) is shown in Fig.10.23

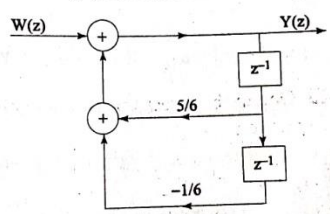
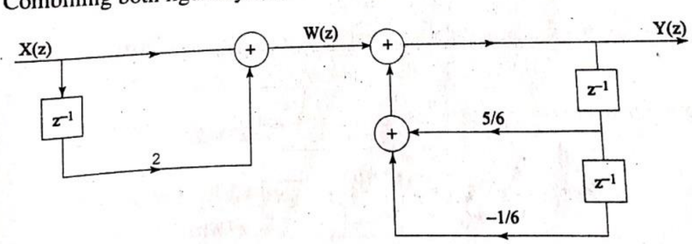


Fig. 10.23 Realization of Eq. (10.125).

Combining both figures yields



Let $\frac{Y(z)}{X(z)} = \frac{Y(z)}{W(z)} \cdot \frac{W(z)}{X(z)}$

where $\frac{W(z)}{X(z)} = \frac{1}{1-\frac{5}{6}z^{-1}+\frac{1}{6}z^{-2}}$

$$\Rightarrow \quad W(z) = X(z) + \frac{5}{6}z^{-1}W(z) - \frac{1}{6}z^{-2}W(z) \qquad (10.133)$$

$$\frac{Y(z)}{W(z)} = 1 + 2z^{-1}$$

$$\Rightarrow Y(z) = W(z) + 2z^{-1}W(z) \qquad (10.134)$$

Realization of Eq. (10.133) is shown in Fig. 10.29.

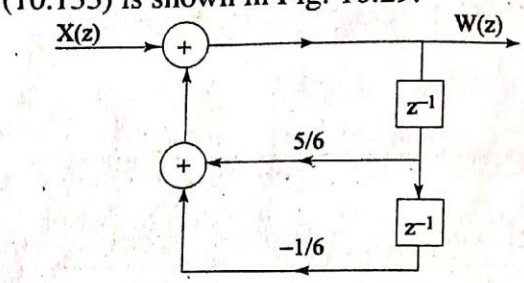


Fig. 10.29 Realization of Eq. (10.133).

Realization of Eq. (10.134) is shown in Fig. 10.30.

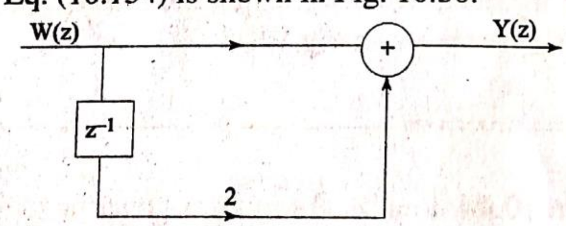


Fig. 10.30 Realization of Eq. (10.134).

Combining Fig. 10.29 and Fig. 10.30 we get the direct form II realization.

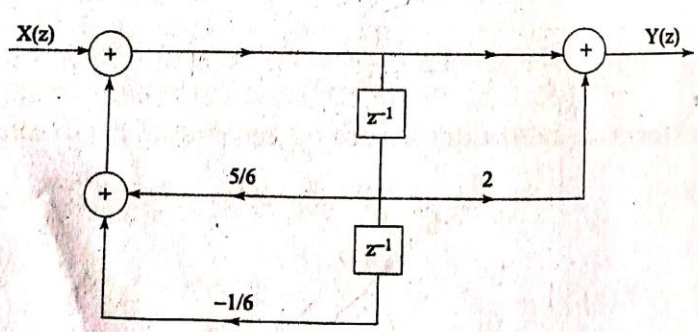


Fig. 10.31 Direct form-II realization of solved problem 10.44.

2	**Obtain the cascade form and parallel form realization of the system described by a difference equation y(n)-1\4y(n-1)-1\8y(n-2)=x(n)+3x(n-1)+2x(n-2)**	**12**	**5**	**K2**

Taking z-transform on both sides, we get

$$Y(z) - \frac{1}{4}z^{-1}Y(z) - \frac{1}{8}z^{-2}Y(z) = X(z) + 3z^{-1}X(z) + 2z^{-2}X(z)$$

$$Y(z)\left[1 - \frac{1}{4}z^{-1} - \frac{1}{8}z^{-2}\right] = X(z)[1 + 3z^{-1} + 2z^{-2}]$$

$$\frac{Y(z)}{X(z)} = \frac{1 + 3z^{-1} + 2z^{-2}}{1 - \frac{1}{4}z^{-1} - \frac{1}{8}z^{-2}}$$

$$= \frac{(1 + z^{-1})(1 + 2z^{-1})}{(1 - \frac{1}{2}z^{-1})(1 + \frac{1}{4}z^{-1})} \quad (10.139)$$

$$= H_1(z)H_2(z)$$

where $$H_1(z) = \frac{1 + z^{-1}}{1 - \frac{1}{2}z^{-1}} \quad (10.140)$$

and

$$H_2(z) = \frac{1 + 2z^{-1}}{1 + \frac{1}{4}z^{-1}} \quad (10.141)$$

The realization of $H_1(z)$ is shown in Fig. 10.33.

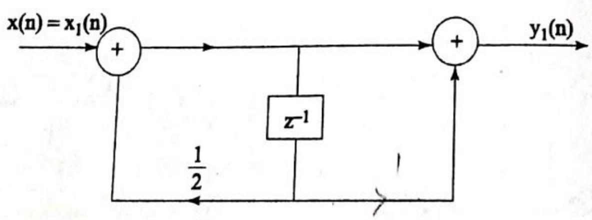


Fig. 10.33 Realization of $H_1(z)$.

The realization of $H_2(z)$ is shown in Fig. 10.34.

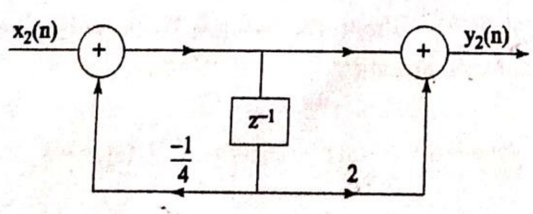


Fig. 10.34 Realization of $H_2(z)$.

Now the cascade realization of $H(z)$ can be obtained by cascading Fig. 10.33 and Fig. 10.34.

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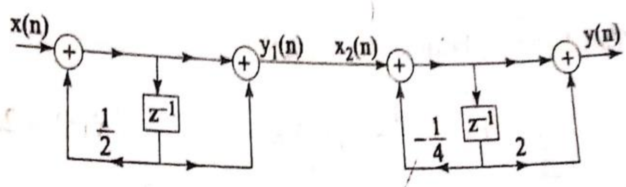

Fig. 10.35 Cascade form realization of solved problem 10.45.

$$y(n) - \frac{1}{4}y(n-1) - \frac{1}{8}y(n-2) = x(n) + 3x(n-1) + 2x(n-2)$$

Taking z-transform on both sides, we get

$$Y(z) - \frac{1}{4}z^{-1}Y(z) - \frac{1}{8}z^{-2}Y(z) = X(z) + 3z^{-1}X(z) + 2z^{-2}X(z)$$

$$\Rightarrow \frac{Y(z)}{X(z)} = \frac{1+3z^{-1}+2z^{-2}}{1-\frac{1}{4}z^{-1}-\frac{1}{8}z^{-2}} \quad (10.144)$$

Now we express the system function in partial fraction form as follows

$$H(z) = 16 + \frac{-15+7z^{-1}}{1+3z^{-1}+2z^{-2}}$$

$$= 16 + \frac{-15+7z^{-1}}{(1+z^{-1})(1+2z^{-1})}$$

$$= 16 + \frac{22}{1+z^{-1}} - \frac{37}{1+2z^{-1}} \quad (10.145)$$

$$\frac{1}{8}z^{-2} - \frac{1}{4}z^{-1} + 1 \;\Big)\; 2z^{-2}+3z^{-1}+1 \;\Big(\; 16$$

$$\underline{2z^{-2} - 4z^{-1} + 16}$$

$$7z^{-1} - 15$$

The realization of Eq. (10.145) is shown in Fig. 10.37.

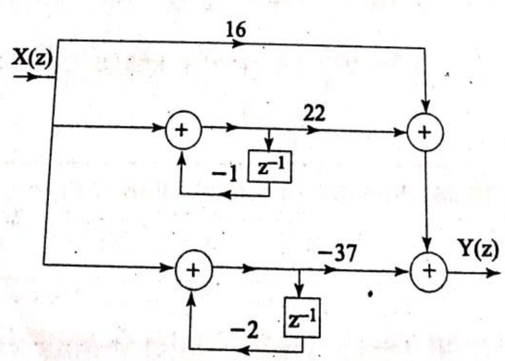

***Fig. 10.37* Parallel-form realization of solved problem 10.45.**

3	**Find the output of y(n)of a LDI discrete time system specified by the equation**	**12**	**5**	**K2**

Y(n)-3\2y(n-1)+1\2y(n-2)-2x(n)+3\2x(n-1)

Solved Problem 10.34 Find the output $y(n)$ of a linear time invariant discrete time system specified by the equation

$$y(n) - \frac{3}{2}y(n-1) + \frac{1}{2}y(n-2) = 2x(n) + \frac{3}{2}x(n-1) \quad (10.110)$$

When the initial condition are $y(-1) = 0, y(-2) = 1$ and the input $x(n) = \left(\frac{1}{4}\right)^n u(n)$

Solution:

Given

$$y(n) - \frac{3}{2}y(n-1) + \frac{1}{2}y(n-2) = 2x(n) + \frac{3}{2}x(n-1)$$

Taking z- transform on both sides, we get

$$Y(z) - \frac{3}{2}[z^{-1}Y(z) + y(-1)] + \frac{1}{2}[z^{-2}Y(z) + z^{-1}y(-1) + y(-2)]$$
$$= 2X(z) + \frac{3}{2}[z^{-1}X(z) + x(-1)] \quad (10.111)$$

We have $\quad y(-1) = 0; \quad y(-2) = 1 \quad \text{and} \quad x(-1) = 0$

Substituting above values in Eq. (10.111), we get

$$Y(z) - \frac{3}{2}z^{-1}Y(z) + \frac{1}{2}\left[z^{-2}Y(z) + 1\right] = X(z)\left[2 + \frac{3}{2}z^{-1}\right]$$

For input $\quad x(n) = \left(\frac{1}{4}\right)^n u(n); \; X(z) = \frac{1}{1 - \frac{1}{4}z^{-1}}$

$$\Rightarrow Y(z)\left[1 - \frac{3}{2}z^{-1} + \frac{1}{2}z^{-2}\right] = -\frac{1}{2} + \frac{1}{1 - \frac{1}{4}z^{-1}}\left(2 + \frac{3}{2}z^{-1}\right)$$

$$Y(z) = \frac{-1}{2\left(1 - \frac{3}{2}z^{-1} + \frac{1}{2}z^{-2}\right)} + \frac{\left(2 + \frac{3}{2}z^{-1}\right)}{\left(1 - \frac{1}{4}z^{-1}\right)\left(1 - \frac{3}{2}z^{-1} + \frac{1}{2}z^{-2}\right)}$$

$$= \frac{-z^2}{2\left(z^2 - \frac{3}{2}z + \frac{1}{2}\right)} + \frac{z^2\left(2z + \frac{3}{2}\right)}{\left(z - \frac{1}{4}\right)\left(z^2 - \frac{3}{2}z + \frac{1}{2}\right)}$$

$$= \frac{-z^2}{2(z-1)\left(z - \frac{1}{2}\right)} + \frac{z^2\left(2z + \frac{3}{2}\right)}{\left(z - \frac{1}{4}\right)(z-1)\left(z - \frac{1}{2}\right)}$$

$$= Y_1(z) \quad + \quad Y_2(z)$$

$$y_1(z) = \frac{-z^2}{2(z-1)\left(z-\frac{1}{2}\right)}$$

$$\frac{y_1(z)}{z} = \frac{-z}{2(z-1)\left(z-\frac{1}{2}\right)}$$

$$= \frac{A}{z-1} + \frac{B}{z-\frac{1}{2}}$$

$$= \frac{-1}{z-1} + \frac{1}{2\left(z-\frac{1}{2}\right)}$$

$$y_1(z) = \frac{-z}{z-1} + \frac{z}{2\left(z-\frac{1}{2}\right)}$$

$$y_1(n) = -u(n) + 0.5\left(\frac{1}{2}\right)^n u(n)$$

$$Y_2(z) = \frac{z^2\left(2z+\frac{3}{2}\right)}{\left(z-\frac{1}{4}\right)(z-1)\left(z-\frac{1}{2}\right)}$$

$$\frac{Y_2(z)}{z} = \frac{z\left(2z+\frac{3}{2}\right)}{\left(z-\frac{1}{4}\right)(z-1)\left(z-\frac{1}{2}\right)}$$

$$= \frac{A_1}{z-\frac{1}{4}} + \frac{B_1}{z-1} + \frac{C_1}{z-\frac{1}{2}}$$

$$= \frac{8}{3\left(z-\frac{1}{4}\right)} + \frac{28}{3(z-1)} - \frac{10}{z-\frac{1}{2}}$$

$$Y_2(z) = \frac{8}{3}\frac{z}{z-\frac{1}{4}} + \frac{28}{3}\frac{z}{z-1} - 10\frac{z}{z-\frac{1}{2}}$$

$$y_2(n) = \frac{8}{3}\left(\frac{1}{4}\right)^n u(n) + \frac{28}{3}u(n) - 10\left(\frac{1}{2}\right)^n u(n)$$

$$A = \cancel{(z-1)} \cdot \frac{-z}{2\,\cancel{(z-1)}\,(z-\frac{1}{2})}\Bigg|_{z=1} = \frac{-1}{2(1-\frac{1}{2})} = -1$$

$$B = \cancel{\left(z-\frac{1}{2}\right)} \frac{-z}{2(z-1)\cancel{\left(z-\frac{1}{2}\right)}}\Bigg|_{z=\frac{1}{2}} = \frac{-\frac{1}{2}}{2(\frac{1}{2}-1)} = \frac{-\frac{1}{2}}{2(-\frac{1}{2})} = \frac{1}{2}$$

$$A_1 = \cancel{\left(z-\frac{1}{4}\right)} \frac{z(2z+\frac{3}{2})}{\cancel{\left(z-\frac{1}{4}\right)}(z-1)\left(z-\frac{1}{2}\right)}\Bigg|_{z=\frac{1}{4}} = \frac{\frac{1}{4}(\frac{1}{2}+\frac{3}{2})}{(\frac{1}{4}-1)(\frac{1}{4}-\frac{1}{2})} = \frac{\frac{1}{4}(2)}{(\frac{-3}{4})(-\frac{1}{4})} = \frac{1}{2}\cdot\frac{16}{3} = \frac{8}{3}$$

$$B_1 = \cancel{(z-1)} \frac{z(2z+\frac{3}{2})}{\cancel{(z-1)}(z-\frac{1}{4})(z-\frac{1}{2})}\Bigg|_{z=1} = \frac{1(2+\frac{3}{2})}{(\frac{3}{4})(\frac{1}{2})} = \frac{\frac{7}{2}}{\frac{3}{8}} = \frac{7}{2}\cdot\frac{8}{3} = \frac{28}{3}$$

$$C_1 = \cancel{\left(z-\frac{1}{2}\right)} \frac{z(2z+\frac{3}{2})}{(z-1)\left(z-\frac{1}{4}\right)\cancel{\left(z-\frac{1}{2}\right)}}\Bigg|_{z=1} = \frac{\frac{1}{2}(1+\frac{3}{2})}{(-\frac{1}{2})(\frac{1}{4})} = \frac{5}{4}\left(\frac{-8}{1}\right) = -10$$

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$$y(n) = \frac{25}{3}u(n) + \frac{8}{3}\left(\frac{1}{4}\right)^n u(n) - \frac{19}{2}\left(\frac{1}{2}\right)^n u(n)$$

4	**Determine the convolution sum of two sequences x(n)={1,4,3,2} h(n)={1,3,2,1}**	**12**	**5**	**K2**

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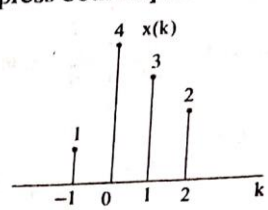
Solved Problem 8.16 Determine the convolution sum of two sequences

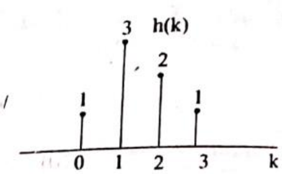
$$x(n) = \{1, \underset{\uparrow}{4}, 3, 2\}; h(n) = \{1, 3, 2, 1\}$$

Solution:

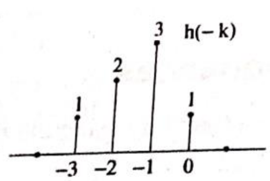
Step 1: The sequence $x(n)$ starts at $n_1 = -1$ and $h(n)$ starts at $n_2 = 0$. Therefore the starting time for evaluating the output sequence $y(n)$ is $n = n_1 + n_2 = -1 + 0 = -1$.

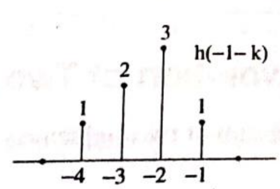
Step 2: Express both sequences in terms the index k.





Step 3: Fold $h(k)$ about $k = 0$, to obtain $h(-k)$





As starting time to evaluate $y(n)$ is -1, shift $h(k)$ by one unit to left to obtain $h(-1-k)$.

$$y(-1) = \sum_{k=-\infty}^{\infty} x(k)h(-1-k)$$

Multiply the two sequences $x(k)$ and $h(-1-k)$ element by element and sum the products

$$y(-1) = \sum_{k=-\infty}^{\infty} x(k)h(-1-k)$$
$$= (0)(1) + (0)(2) + 0(3) + (1)(1) + 4(0) + 3(0) + 2(0) = 1$$

Increment the index by 1, i.e., n to zero, shift the sequence to right to obtain $h(-k)$ and then multiply the two sequences $x(k)$ and $h(-k)$ element by element and sum the products we get

$$y(0) = \sum_{k=-\infty}^{\infty} x(k)h(-k)$$
$$= (0)(1) + (0)(2) + 1(3) + 4(1) + 3(0) + 2(0) = 7$$

Similarly,

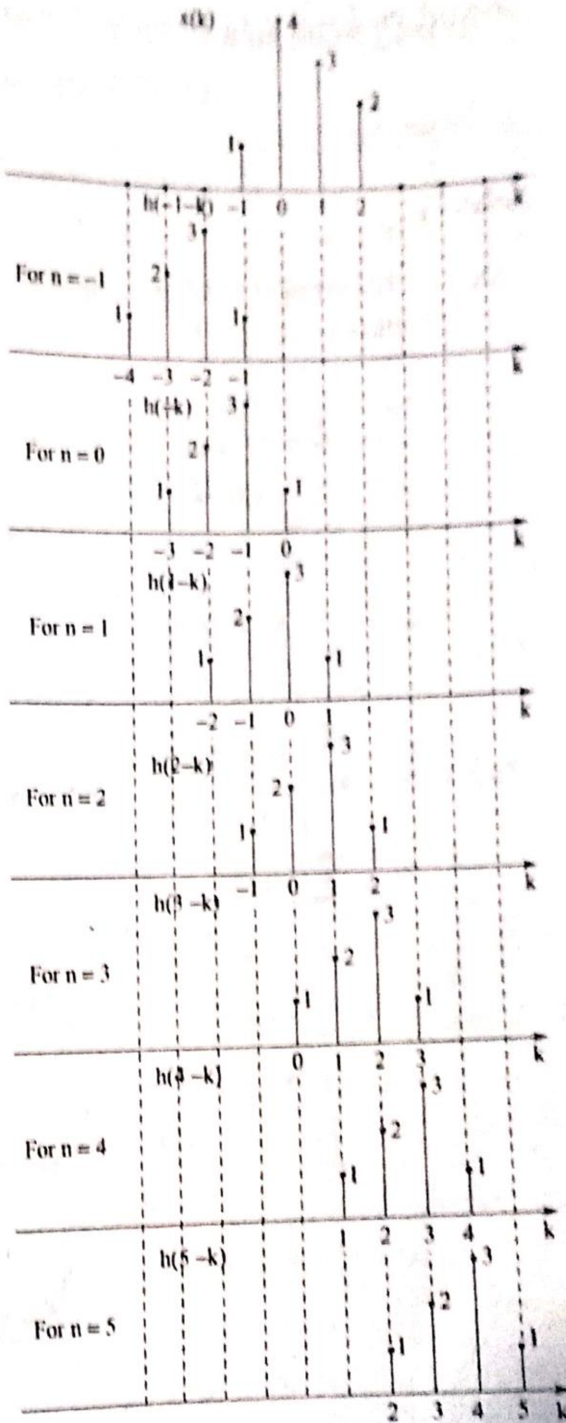
$$y(1) = \sum_{k=-\infty}^{\infty} x(k)h(1-k)$$
$$= 0(1) + 1(2) + 4(3) + 3(1) + 2(0) = 17$$

$$y(2) = \sum_{k=-\infty}^{\infty} x(k)h(2-k)$$
$$= 1(1) + 4(2) + 3(3) + 2(1) = 20$$

$$y(3) = \sum_{k=} x(k)h(3-k)$$
$$= 1(0) + 4(1) + 3(2) + 2(3) + 0(1) = 16$$

$$y(4) = \sum_{k=-\infty}^{\infty} x(k)h(4-k)$$
$$= 3.1 + 2.2 = 7$$

$$y(5) = \sum_{k=-\infty}^{\infty} x(k)h(5-k) = 2.1 = 2$$



y(n) = {1, 7, 17, 20, 16, 7, 2}
↑

Fig. 8.9

For remaining values of n the value $y(n) = 0$. Therefore we stop at $n = 5$.

5	**Find the convolution of the following sequence h(n)=3δ(n-1)+4 δ(n-2)+2 δ(n-3)** **X(n)=2 δ(n+1)- δ(n)+ δ(n-1)+3 δ(n-2)**	**12**	**5**	**K2**

Solved Problem 8.20 Find the convolution of the following sequence

$$x(n) = 2\delta(n+1) - \delta(n) + \delta(n-1) + 3\delta(n-2)$$
$$h(n) = 3\delta(n-1) + 4\delta(n-2) + 2\delta(n-3)$$

Solution:

Given

$$x(n) = \{2 \; -\underset{\uparrow}{1}, 1, 3\}; \quad n_1 = -1$$
$$h(n) = \{\;3, 4, 2\}; \quad n_2 = 1$$

Method 1

$$n_1 + n_2 = 0.$$

Therefore, the sequence $y(n)$ starts at $n = 0$. By following the steps given for matrix convolution, we obtain

$h(n)$ \ $x(n)$	2	−1	1	3
3	6	−3	3	9
4	8	−4	4	12
2	4	−2	2	6

$$= 6, 8-3, 4-4+3, -2+4+9, 12+2, 6$$
$$y(n) = \{\underset{\uparrow}{6}, 5, 3, 11, 14, 6\}$$

Method 2

Given the sequences

$$x(n) = \{2, -1, 1, 3\}; h(n) = \{3, 4, 2\}$$

Writing the sequences in matrix form and multiplying we get

$$\begin{bmatrix} 3 & 0 & 0 & 0 \\ 4 & 3 & 0 & 0 \\ 2 & 4 & 3 & 0 \\ 0 & 2 & 4 & 3 \\ 0 & 0 & 2 & 4 \\ 0 & 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 6 \\ 5 \\ 3 \\ 11 \\ 14 \\ 6 \end{bmatrix}$$

$$y(n) = \{6, 5, 3, 11, 14, 6\}$$

	Part C(20 Mark Questions with Key)			
1	**Find the impulse response and step response for the following system** **Y(n)-3\4y(n-1)+1\8y(n-2) = x(n)**	20	5	K2

Solution:

Given

$-\frac{3}{4} z^{-1} Y(z)$

$$y(n) - \frac{3}{4}y(n-1) + \frac{1}{8}y(n-2) = x(n)$$

Taking z-transform on both sides, we get

$$Y(z) - \frac{3}{4}\left[z^{-1}Y(z) + y(-1)\right] + \frac{1}{8}\left[z^{-2}Y(z) + z^{-1}y(-1) + y(-2)\right] = X(z)$$

Substituting initial conditions $y(-1) = y(-2) = 0$ yields

$$Y(z) - \frac{3}{4}z^{-1}Y(z) + \frac{1}{8}z^{-2}Y(z) = X(z)$$

$$\Rightarrow \quad \frac{Y(z)}{X(z)} = \frac{1}{1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}}$$

Impulse response for $x(n) = \delta(n)$; $X(z) = 1$

$$\Rightarrow \quad Y(z) = \frac{1}{1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}} = \frac{z^2}{z^2 - \frac{3}{4}z + \frac{1}{8}}$$

$$\frac{Y(z)}{z} = \frac{z}{\left(z - \frac{1}{2}\right)\left(z - \frac{1}{4}\right)} = \frac{A}{z - \frac{1}{2}} + \frac{B}{z - \frac{1}{4}}$$

$$A = \left(z - \frac{1}{2}\right) \frac{z}{\left(z - \frac{1}{2}\right)\left(z - \frac{1}{4}\right)}\Bigg|_{z=\frac{1}{2}} = \frac{\left(\frac{1}{2}\right)}{\frac{1}{2} - \frac{1}{4}} = 2$$

$$B = \left(z - \frac{1}{4}\right) \frac{z}{\left(z - \frac{1}{2}\right)\left(z - \frac{1}{4}\right)}\Bigg|_{z=\frac{1}{4}} = \frac{\frac{1}{4}}{\left(\frac{1}{4} - \frac{1}{2}\right)} = -1$$

$$\Rightarrow \frac{Y(z)}{z} = \frac{2}{z - \frac{1}{2}} - \frac{1}{z - \frac{1}{4}}$$

$$Y(z) = 2\frac{z}{z - \frac{1}{2}} - \frac{z}{z - \frac{1}{4}}$$

Taking inverse z-transform we get $y(n) = 2\left(\frac{1}{2}\right)^n u(n) - \left(\frac{1}{4}\right)^n u(n)$

Step response For a unit step input $x(n) = u(n);\ X(z) = \frac{z}{z-1}$

$$\frac{Y(z)}{X(z)} = \frac{1}{1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}}$$

$$Y(z) = \frac{z}{z-1} \quad \frac{z^2}{z^2 - \frac{3}{4}z + \frac{1}{8}}$$

$$\frac{Y(z)}{z} = \frac{z^2}{(z-1)\left(z - \frac{1}{2}\right)\left(z - \frac{1}{4}\right)}$$

$$= \frac{A}{z-1} + \frac{B}{z - \frac{1}{2}} + \frac{C}{z - \frac{1}{4}}$$

$$A = (z-1) \frac{z^2}{(z-1)\left(z - \frac{1}{2}\right)\left(z - \frac{1}{4}\right)}\Bigg|_{z=1}$$

$$= \frac{(1)^2}{\left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{4}\right)} = \frac{1}{\left(\frac{1}{2}\right)\left(\frac{3}{4}\right)} = \frac{8}{3}$$

$$B = \left(z - \frac{1}{2}\right) \frac{z^2}{(z-1)\left(z - \frac{1}{2}\right)\left(z - \frac{1}{4}\right)}\Bigg|_{z=\frac{1}{2}}$$

$$= \frac{\left(\frac{1}{2}\right)^2}{\left(\frac{1}{2} - 1\right)\left(\frac{1}{2} - \frac{1}{4}\right)} = \frac{\frac{1}{4}}{\left(-\frac{1}{2}\right)\left(\frac{1}{4}\right)} = -2$$

$\frac{1}{4} \times -8$

	$C=\left(z-\frac{1}{4}\right)\frac{z^2}{(z-1)\left(z-\frac{1}{2}\right)\left(z-\frac{1}{4}\right)}\Bigg\vert_{z=\frac{1}{4}}$ $=\frac{\left(\frac{1}{4}\right)^2}{\left(\frac{1}{4}-1\right)\left(\frac{1}{4}-\frac{1}{2}\right)}=\frac{\frac{1}{16}}{\left(\frac{-3}{4}\right)\left(\frac{-1}{4}\right)}=\frac{1}{3}$ $\frac{Y(z)}{z}=\frac{8}{3(z-1)}-\frac{2}{z-\frac{1}{2}}+\frac{1}{3\left(z-\frac{1}{4}\right)}$ $Y(z)=\frac{8}{3}\cdot\frac{z}{z-1}-2\frac{z}{z-\frac{1}{2}}+\frac{1}{3}\quad\frac{z}{z-\frac{1}{4}}$ Taking inverse $-z$ transform yields $y(n)=\frac{8}{3}u(n)-2\left(\frac{1}{2}\right)^n u(n)+\frac{1}{3}\left(\frac{1}{4}\right)^n u(n)$			
2	**Determine the impulse response h(n)for the system de-scribed by the second order difference equation** **Y(n)=0.6y(n-1)-0.8y(n-2)+x(n)**	20	5	K2

Solved Problem 8.4 Determine the impulse response $h(n)$ for the system described by the second-order difference equation

$$y(n) = 0.6y(n-1) - 0.08y(n-2) + x(n)$$

Solution:

Given
$$y(n) = 0.6y(n-1) - 0.08y(n-2) + x(n)$$

We know the total response

$$y(n) = y_h(n) + y_p(n). \tag{8.35}$$

For impulse $x(n) = \delta(n)$, the particular solution

$$y_p(n) = 0$$
$$\Rightarrow \quad y(n) = y_h(n).$$

The homogeneous solution can be found by substituting $x(n) = 0$

$$\Rightarrow \quad y(n) - 0.6y(n-1) + 0.08y(n-2) = 0 \tag{8.36}$$

Let the solution be
$$y_h(n) = \lambda^n \tag{8.37}$$

Substituting Eq. (8.37) in Eq. (8.36) we obtain

$$\lambda^n - 0.6\lambda^{n-1} + 0.08\lambda^{n-2} = 0$$
$$\lambda^{n-2}[\lambda^2 - 0.6\lambda + 0.08] = 0$$
$$\Rightarrow \quad \lambda^2 - 0.6\lambda + 0.08 = 0$$

The roots of the characteristic equation are

$$\lambda_1 = 0.4; \lambda_2 = 0.2$$

The general form of the solution of the homogeneous equation is

$$y_h(n) = c_1\lambda_1^n + c_2\lambda_2^n$$
$$= c_1(0.4)^n + c_2(0.2)^n \tag{8.38}$$

From Eq. (8.38)

$$y(0) = c_1 + c_2$$
$$y(1) = 0.4c_1 + 0.2c_2 \tag{8.39}$$

From the difference equation we have

$$y(0) = 0.6y(-1) - 0.08y(-2) + x(0)$$

$$= 1 \qquad \boxed{\begin{matrix} y(-1) = y(-2) = 0 \\ x(0) = \delta(0) = 1 \end{matrix}}$$

$$y(1) = 0.6y(0) - 0.08y(-1) + x(1)$$
$$= 0.6(1) - 0.08(0) + 0$$
$$= 0.6$$

$$\Rightarrow \quad y(0) = 1$$
$$y(1) = 0.6$$

Comparing Eq. (8.39) and Eq. (8.40)

$$c_1 + c_2 = 1$$
$$0.4c_1 + 0.2c_2 = 0.6$$

and solving for c_1 and c_2 we get

$$c_1 = 2$$
$$c_2 = -1$$

Substituting the values in Eq. (8.38) yields

$$y(n) = 2(0.4)^n u(n) - (0.2)^n u(n)$$